

Exam Calculus 2

Friday, June 2, 2006, 9:00-12:00

Total 5 exercises and: 100 points

Exercise 1 [20 points] The curve K is defined by the following parametrization

$$\begin{cases} x(t) = t^3 - 3t \\ y(t) = 4 - t^2 \end{cases}, t \in [-\sqrt{3}, \sqrt{3}]$$

i) [5 points] Find the coordinates of the points at which K has horizontal or vertical tangents.

ii) [5 points] Show that K is a closed curve, by showing that at $t = -\sqrt{3}$ and $t = \sqrt{3}$ the curve intersects itself.

iii) [5 points] Sketch the graph of K using the information from *i)* and *ii)*.

iv) [5 points] Compute the area enclosed by the curve K .

Exercise 2 [20 points] Let: $f(x, y) = x \log(y)$, and: $x(u, v) = u + v$, and :
 $y(u, v) = u - v$. Find the partial derivatives $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$ when function g
is defined by: $g(u, v) = f(x(u, v), y(u, v))$.

Exercise 3 [20 points] Let function f be defined as: $f(x, y) = 8 - x^2 - y^2$.
Find the equation of the **normal line** to f at $P(1, 1, 6)$.

Exercise 4 [20 points]

Let D be a square defined by: $D = \{(x, y) \in \mathbb{R}^2 \mid |x| + |y| \leq 5\}$.

Questions

- a. [10 pts] Make a sketch of D in the x - y -plane.
- b. [10 pts] Compute : $\iint_D (y + 1).dA$.

Exercise 5 [20 points]

Consider the function:

$$\begin{cases} f(x) = (1 - x^2)/5, x \in [0,1] \\ f(x) = 0, x \notin [0,1] \end{cases}$$

Now consider the lens-shaped solid S obtained by rotation of the area between: $f(x)$ and: $-f(x)$ for: $x \in [0,1]$ about the y -axis.

Questions

- a. make a sketch of solid S .
- b. Compute the volume of the solid S using the “cylindrical shells method” (the summation of thin concentric cylinders that are symmetric about the y -axis).
- c. Compute the volume of the solid S by slicing (slice the solid into slices that are perpendicular to the y -axis and integrate)
- d. Explain the *similarity* or *difference* between your answers to **5b** and **5c**.

Good Luck !!!