

Toets Neurale Netwerken en Fuzzy Logic
28 juni 2002

Instructions: Please write clearly, and *justify your computations and conclusions as much as possible*. Good luck!

(Obvious, but still...) *Je kan natuurlijk in het Nederlands je antwoorden geven!*

1. The input correlation matrix $R = E[zz^T]$, the cross-correlation vector $h = E[tz]$ and the expectation $c = E[t^2]$ of the squared targets for an ADALINE with two inputs and zero bias are respectively

$$R = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
$$h = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$
$$c = 42$$

- (a) Write down the expression for the performance function corresponding to such matrices as a function of the two weights w_1 and w_2 .
- (b) Does the performance function have a unique minimum? If yes, why is it so, and what are the optimal weights (w_1^*, w_2^*) in such case? If no, why is it so?
- (b) Write down the LMS learning rule for this case.
2. Let $U = \{2, 12\}$ and $V = \{1, 7, 13\}$ be the universes of discourse. The fuzzy sets $A, B \subset U \times V$ are defined from the following matrices:

$$A \rightarrow \begin{array}{ccc} & v_1 & v_2 & v_3 \\ u_1 & 0.9 & 0.4 & 0.1 \\ u_2 & 0.1 & 0.4 & 0.9 \end{array}$$
$$B \rightarrow \begin{array}{ccc} & v_1 & v_2 & v_3 \\ u_1 & 0 & 0.6 & 1 \\ u_2 & 0 & 0 & 0.3 \end{array}$$

Compute the fuzzy sets $A \cup B$, $A \cap B$.

3. Consider an instar neuron with 3 inputs. Assume that such network has weights $w_{11} = -0.3333$, $w_{12} = 0.7778$, $w_{13} = 0.5329$ and bias equal to zero. Let p be an input vector of unit length; how close must p be to the weight vector, so that the output will be a 1?
4. A multilayer network with three layers is given as in the figure below.

Assume that the transfer functions f_i are all linear, $f_i(x) = x$ for $i = 1, 2, 3$.

- (a) Prove that this network is equivalent (in the sense that it implements the same map between inputs and outputs) to a *single layer* network. What is the weight matrix of such single layer network, as a function of the weight matrices W_i of the three-layer one?
 - (b) Assume that the backpropagation algorithm is used to train this network. To which other learning rule does such training technique reduce to in this particular case?
5. An associative network with two inputs is trained with Hebb's learning rule, with learning rate $\alpha = 1$. The initial weights are $w^0 = 1$ (for the unconditioned stimulus p^0) and $w(0) = 0$ (for the conditioned stimulus p).

Assume that the learning sequence is

$$\{p^0(1) = 0, p(1) = 1\}, \{p^0(2) = 1, p(2) = 1\}, \{p^0(3) = 0, p(3) = 1\}, \{p^0(2) = 1, p(2) = 1\}, \dots$$

- (a) Compute the first two iterations of the learning process.
- (b) Assume that the learning process continues indefinitely with the same sequence as above. What are the consequences for the weight w ?
- (c) Discuss ways to correct the situation you found in point (b) above.